FINAL: BIII REPRESENTATION THEORY

Date: $25^{\mathbf{th}}$ **November 2022**

The Total points is **110** and the maximum you can score is **100** points.

A **representation** would mean a **representation on a vector space over complex numbers**.

(1) (15 points) Let $\rho : G \to GL(V)$ be a faithful G-representation. For $g \in G$, show that the map $rho(g)$ is G-equivariant iff g is in the center $Z(G)$.

(2) (5+15=20 points) Define the standard representation of S_n. Let V be the standard representation of S_4. Decompose $V \otimes V \otimes V$ as direct sum of irreducible representation.

(3) (25 points) Let V be the standard representation of S_3. Let $G = S_3 \times S_3$, H be the subgroup $S_3 \times \{e\}$ and D be the subgroup $\{(\sigma, \sigma) : \sigma \in S_3\}$ of G. Note that D and H are isomorphic to S_3. Decompose $W_1 = \mathrm{Ind}_H^G V$ and $W_2 = \mathrm{Ind}_D^G V$ as direct sum of irreducible G-representations.

(4) (25 points) Let G be finite group and V be a G representation and χ_V its character. Let $R = End_G(V)$ be the ring of endomorphism of G-equivariant linear map from V to itself. Show that R is abelian iff $(\chi_V, \chi) = 1$ for every irreducible character χ of G.

(5) (25 points) Compute the number of irreducible representations of the alternating group A_7, dihedral group D_7 and $A_7 \times D_7$. Also compute dimensions of each of them.